(Formal) Software Verification via Logic

(using One-Counter Automata)

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1. Formal Verification

Motivation

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def dummy(x:int,y:int):
    z = 0
    if x>0 & y>0:
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    return z
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"Program testing can be a very effective way to show the presence of bugs, but it is hopelessly inadequate for showing their absence." - E. Dijkstra

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- A lot more at `https://en.wikipedia.org/wiki/List_of_software_bugs`

Formal Verification

Formal Verification

Formal verification is the process of proving the correctness of intended algorithms underlying a system with respect to a certain formal specification or property, using formal methods of mathematics.

Formal Verification

Formal Models

- Finite State Machines
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Advantages of Formal Verification

- Formally proving correctness and ensure safety
- Significantly reduces the verification time

2. One-Counter Automata

Using the control flow graph (CFG)

```
1  skip = 2
2  retake = 3
3  retake += skip
4  while retake >= 0:
5      if retake == 3:
6          print("You get a reminder")
7      if retake == 4:
8          print("You get soft warning")
9      if retake == 5:
10         print("You get hard warning")
11     if retake >= 6:
12         assert("God forbid!")
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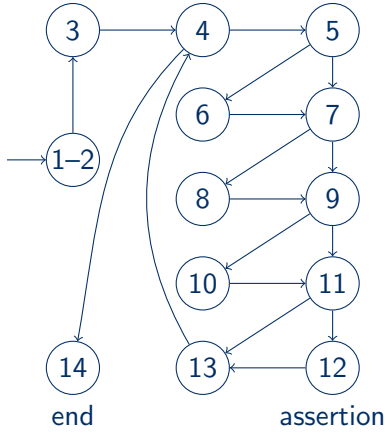

Extending the CFG with a counter

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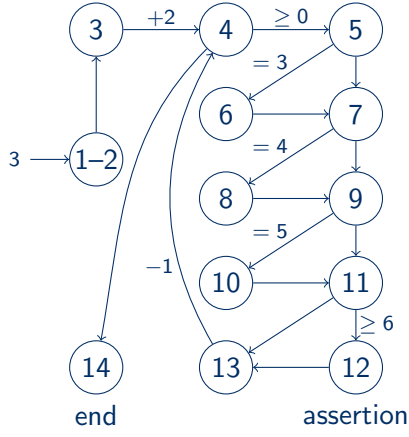

Counter : = retake

Parametric one-counter automata

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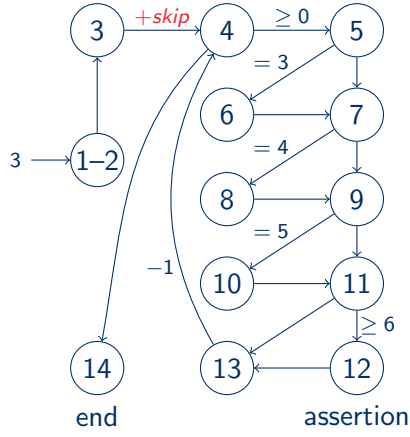

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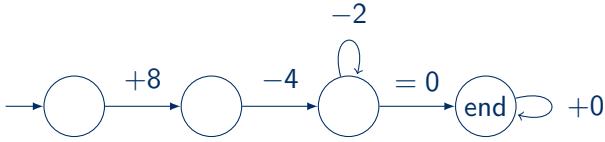
(Parametric) One-Counter Automata



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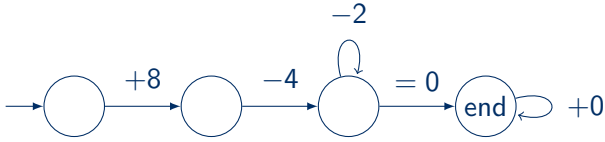
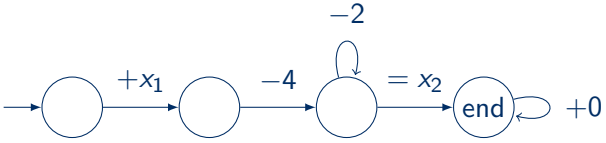


Counter Value has to be non-negative all the time!

(Parametric) One-Counter Automata

Natural-valued parameters

$X = \{x_1, \ldots, x_n\}$



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Decidability Questions

Definition (Parameter-value Reachability)

Is there some valuation $V : X \to \mathbb{N}$ such that there is some run of $\mathcal{A}$ that reaches/avoids a good/bad state?

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Non-parametric Versions of the above also

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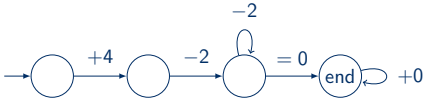

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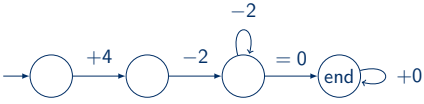

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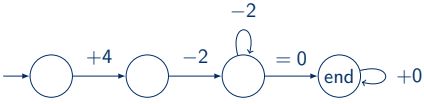

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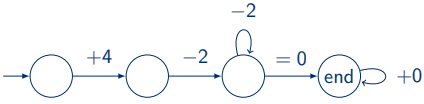

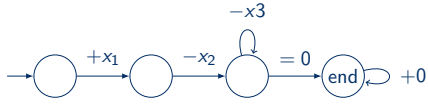

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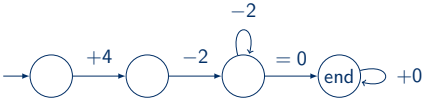

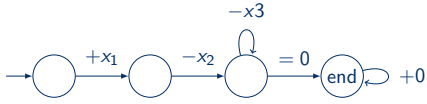

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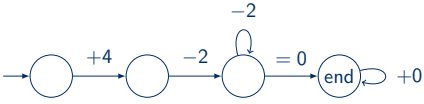

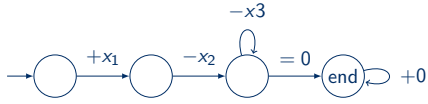

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Presburger Arithmetic with divisibility:
$PA\ +\ |$
$(a \mid b \iff \exists c \in \mathbb{Z} : b = ac)$

Complexity and Decidability

- Non-parametric Reachability: **NP** (Presburger Arithmetic/PA)
- Non-parametric Synthesis: **coNP** (Reduction complement to Non-parametric Reach)
- Parametric Reachability: **NEXP** (Existential PAD)
- Parametric Synthesis: **N2EXP** (BIL : a fragment of one-alternation PAD)

4. Conclusion

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Research:

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- Markov Decision Process
- Hybrid Automata